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Formulae for study of LID induced diffusion in CP star model atmospheres

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Abstract. Formulae suitable for computing LID acceleration and corresponding diffusional segregation of isotopes in CP star model atmospheres are given.

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Light-induced drift (LID) as a phenomenon responsible for separation of isotopes in the atmospheres of CP stars has been proposed by Atutov and Shalagin (1988). Thereafter we have studied evolutionary abundance changes of Hg and its isotopes (see Sapar *et al.*, C_Sapar, these proceedings). Here we give formulae describing diffusion of isotopes in the form best suited for numerical model computations.

Diffusive transfer of radiation holds in the deeper layers of stellar atmospheres, where monochromatic radiative flux can be described by formula

$$F_\nu = F K_R \frac{dB_\nu}{\kappa_\nu dT}\,, \qquad \text{where } \frac{1}{K_R} = \int_0^\infty \frac{dB_\nu}{\kappa_\nu dT} d\nu.$$

Here F is the total radiation flux and K_R is the Rosseland opacity integral. In the opacity coefficient κ_ν the overlapping spectral lines of the trace element studied play an important role, and thus $\kappa_\nu = c_\nu + \sum_j \sigma_j W_j(u_\nu)$, where c_ν is the continuous opacity coefficient and σ_j is the transition cross-section per gram. Summation is made over spectral lines j with line profile functions W_j, usually being the Voigt functions with argument given by $u_\nu = (\nu - \nu_j)/\Delta\nu_T$, where $\Delta\nu_T$ is the thermal Doppler width of the spectral line. As we have shown, the effective acceleration of LID due to spectral line j can be expressed quite similarly to the usual expression of radiative acceleration

$$a_j^L = \frac{\pi\varsigma_j}{c} \int_0^\infty \frac{\partial W_j(u_\nu)}{\partial u_\nu} F_\nu d\nu, \qquad \varsigma_j = q\epsilon\sigma_j, \tag{1}$$

where $q = M v_T c/2h\nu$ and the efficiency of LID is $\epsilon = (C_u - C_l)/(A_u + C_u)$. Here C_u and C_l are the collision rates of particles in upper and lower states, respectively, and A_u is the probability of spontaneous transitions.

Expression (1) in the region of diffusive transfer of radiation takes the form

$$a_j^L = \frac{\pi\varsigma_j F K_R}{c} \int_0^\infty \frac{\partial W_j(u_\nu)}{\partial u_\nu} \frac{dB_\nu}{\kappa_\nu dT} d\nu. \tag{2}$$

Both isotopic and hyperfine splitting of spectral lines of all ions should be taken into account to calculate accelerations producing segregation of isotopes.

The equation of continuity for isotope i in the plane-parallel stellar atmosphere has the form $d\rho_i/dt + d(\rho_i V_i)/dr = 0$. The model atmosphere data correspond to standard points, being equidistant on the logarithmic scale of mean optical depth. These points are enumerated as layers n growing downwards. Treating n as a continuous parameter, we change variables in equation of continuity. Since $\frac{d}{dr} = \frac{d/dn}{dr/dn}$ and $\rho\frac{dr}{dn} = -\frac{d\mu}{dn}$, where μ is total column density, we get for radial gradient

$$\frac{d}{dr} = -\gamma\frac{d}{dn}, \quad \text{where } \gamma = \frac{\rho}{d\mu/dn} = \frac{\rho}{\mu d\ln\mu/dn} \ .$$

Denoting a ratio of current concentration to its initial value as C_i, we can write $\rho_i = \rho_i^0 C_i$ and the equation of continuity reduces to

$$\frac{d\ln C_i}{dt} = \frac{\gamma}{\rho_i}\frac{d(\rho_i V_i)}{dn} \ . \tag{3}$$

Logarithms are used to avoid possible negative values of C_i in time integration. Diffusion velocity V_i in the presence of stellar wind can be found from equation

$$\rho_i V_i = \rho_i(a_i - g)t - \Delta\frac{d\rho_i}{dr} \ , \qquad \frac{d\rho_i}{dr} = -\gamma\frac{d\rho_i}{dn}, \tag{4}$$

where a_i is the sum of radiative and LID accelerations, t is the mean free flight time of the particles , m is the mean mass of buffer particles and $\Delta = kTt/m$ is the diffusion coefficient of trace particles. Thus from equation (4) we find

$$\frac{V_i}{\gamma\Delta} = \frac{m(a_i - g)}{kT\gamma} + \frac{d\ln(\rho_i^0 C_i)}{dn} \quad \text{and} \quad \frac{V_i}{\gamma\Delta} = \frac{ma_i}{kT\gamma} + \frac{d\ln C_i}{dn}. \tag{5}$$

Here the last expression has been obtained from the first taking into account that for model stellar atmospheres approximately holds $mg/kT\gamma = d\ln\rho_i^0/dn$. The velocity V_i is to be used in the equation of continuity (3), reducing it to a generalized Fokker–Planck equation. The equation (5) can be used also for a crude prediction of concentrations C_i. For the final equilibrium state in the case of no stellar wind $V_i = 0$ and in the case of constant mass loss rate $\rho_i V_i = \dot{m}_i$.

As we see, for evolutionary computations we need to find derivatives $d\ln\rho/dn$, $d\mu/dn$ and $d\gamma/dn$, which correspond to buffer gases and several derivatives for each isotope of the trace (impurity) particles. The derivatives have been found using the 4th order Lagrange interpolation formulae for equidistant nodes.

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References

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